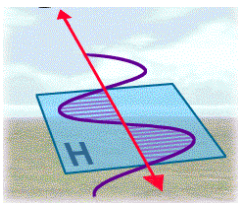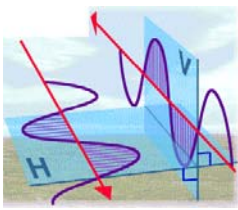# 6. ENVISAT/ASAR DUAL POLARISATION CASE

6.1 ASAR dual polarization data formats
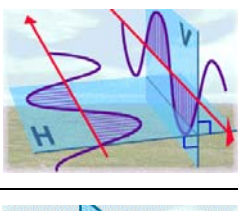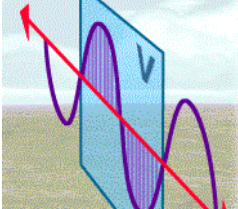
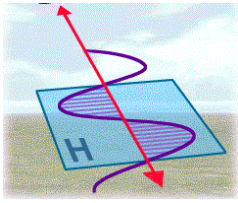### 6.1.1 Supported ASAR data formats
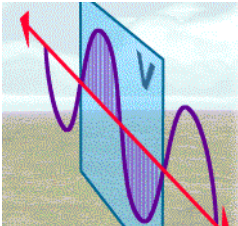
6.1.1.1 Alternating Polarization modes

The ASAR instrument may be configured to acquire data according to one of the Alternating Polarization (AP) modes.

Classical fully polarimetric SAR devices perform data acquisition with two emission and reception polarization channels. ASAR, instead, emits signals on a single polarization channel (H or V) while reception is performed on one or two orthogonal polarizations (H and/or V).

The different possible configurations are summarized in the following figure.

PP configuration	Received polarizations	Emitted polarization	
PP 1	H	H	
	V		
PP2	H	V	
	V		

PP3	H	H	
	V	V	

where PP stands for Partial Polarization or Partially Polarimetric.

6.1.1.2   Supported AP data formats

Partial polarization data sets are delivered under different formats adapted to specific types of application.

Among the different AP data formats, three are supported by PolSARpro and can be exploited to provide an interpretation of the polarimetric properties of scattering.

- Alternating Polarization mode Single-look complex (APS) data format

Data type	Sets of **complex** scattering coefficients corresponding to the selected PP configuration
Geometry	**Slant Range**
Coverage	100 km along-track, 56 - 100 km across-track
Pixel spacing	**Equal to Resolution** : e.g 7.8m * 4m

- Alternating Polarization mode Precision (APP) image format

Data type	Images of **real reflectivity** corresponding to the selected PP configuration
Geometry	**Ground Range**
Coverage	100 km along-track, 56 - 100 km across-track
Pixel spacing	12.5 m * 12.5 m

- ASAR Alternating Polarization mode ellipsoid Geocoded (APG) image

Data type	Images of **real reflectivity** corresponding to the selected PP configuration
Geometry	**Geocoded Ground Range**
Coverage	100 km * 100 km
Pixel spacing	12.5 m * 12.5 m

6.1.2 Relation with fully polarimetric representations

Fully polarimetric quantities are measured from two acquisitions performed with orthogonal polarization states (generally H and V) in order to form each pixel scattering matrix [S]

The coherent scattering matrix modifies the polarization of an incident wave according to the following expression

$$\underline{E}_s = [S]\underline{E}_i \tag{1}$$

The scattering matrix is built from two acquisitions as follows

$$[S] = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} = \begin{bmatrix} \underline{E}_{sh} & \underline{E}_{sv} \end{bmatrix}$$

$$\text{with } \underline{E}_{sh} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} S_{hh} \\ S_{vh} \end{bmatrix} \text{ and } \underline{E}_{sv} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S_{hv} \\ S_{vv} \end{bmatrix} \tag{2}$$

where $\underline{E}_{sh}$ and $\underline{E}_{sv}$ correspond to the scene response to normalized incident horizontally and vertically polarized waves respectively.

6.1.2.1 APS format

APS format images are composed of coherent scattering coefficients and my then be expressed directly from the expression given in (2) according to the selected PP configuration

- PP1

A pixel is represented by a Jones vectors formed with two of the scattering matrix coefficients

$$\underline{E}_{PP1} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} S_{hh} \\ S_{vh} \end{bmatrix} \tag{3}$$

- PP2

A pixel is represented by a Jones vectors formed with two of the scattering matrix coefficients

$$\underline{E}_{PP2} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} S_{hv} \\ S_{vv} \end{bmatrix} \tag{4}$$

- PP3

A pixel is represented by a two element complex vector formed with two of the scattering matrix coefficients

$$\underline{E}_{PP3} = \begin{bmatrix} \begin{bmatrix} S_{hh} & S_{hv} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} S_{hh} \\ S_{vv} \end{bmatrix} \tag{5}$$

One may note that, oppositely to the vector $\underline{E}_s$, the Jones vectors $\underline{E}_{sh}$ and $\underline{E}_{sv}$ are received in a polarization basis.

It is then possible to represent $\underline{E}_{sh}$ and $\underline{E}_{sv}$ into any desired polarization basis using classical Special Unitary transformation matrices

$$\underline{E}' = [U(\phi, \tau, \alpha)] \underline{E} \tag{6}$$

Such a transformation may also be applied to $\underline{E}_s$ in PP3 configuration, but without the physical interpretation associated to the formalism of changes of polarization basis. This might be particularly useful to perform a transformation similar to the one linking [C] and [T] matrices

$$\underline{E}_s' = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \underline{E}_s = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{hh} + S_{vv} \\ S_{vv} - S_{hh} \end{bmatrix} \tag{7}$$

6.1.2.2   APP and APG formats

APP and APG format images are composed of intensities and my then be expressed from the square moduli of the Sinclair matrix elements, according to the selected PP configuration

- PP1

$$\underline{I}_{PP1} = \begin{bmatrix} |S_{hh}|^2 \\ |S_{vh}|^2 \end{bmatrix} \tag{8}$$

- PP2

$$\underline{I}_{PP2} = \begin{bmatrix} |S_{vv}|^2 \\ |S_{hv}|^2 \end{bmatrix} \tag{9}$$

- PP3

$$\underline{I}_{PP3} = \begin{bmatrix} |S_{hh}|^2 \\ |S_{vv}|^2 \end{bmatrix} \qquad (10)$$

One may note that APP and APG mode data do not exactly correspond to intensities derived from equivalent APS data, due to the slant range to ground range mapping transformation.

6.2   Speckle filtering

6.2.1   APP and APG format data filtering

As it was seen in the former paragraph, APP and APG format data consist of incoherent intensity values that are fit to incoherent speckle filtering.

The formulation of the boxcar and Lee filters can be adapted to intensity variables.

6.2.1.1   <u>APP and APG boxcar filter</u>

Filtered intensity vector estimates, $\tilde{I}_{PPi}$, are constructed by computing the sample mean over each pixel neighborhood, defined by a sliding window of $(N_w \times N_w)$ pixels

$$\tilde{I}_{PPi} = \left\langle I_{PPi} \right\rangle_{N_w} \qquad (11)$$

6.2.1.2   <u>APP and APG Lee filter</u>

The adaptation of Lee MMSE filter to APP and APG data formats leads to the following intensity vector estimate expression

$$\tilde{\underline{I}}_{PPi} = \left\langle \underline{I}_{PPi} \right\rangle_{N_w} + k\left(\underline{I}_{PPi} - \left\langle \underline{I}_{PPi} \right\rangle_{N_w} \right) \qquad (12)$$

where k is an adaptive filtering coefficient based on local statistics and given by

$$k = \frac{\mathrm{var}\langle I \rangle - E^2(\langle I \rangle)\sigma_n^2}{\mathrm{var}\langle I \rangle [1 + \sigma_n^2]}$$

with $\sigma_n^2 = \dfrac{1}{L}$ the a priori speckle variance

$$\qquad (13)$$

where I represents the sum of the components of the vector $\underline{I}_{PPi}$.

The two components of the intensity vector $\underline{I}_{PPi}$ may be filtered independently by applying separate scalar filters.

6.2.2   APS format data filtering

APS format data consist of coherent scattering coefficients that cannot be directly used as inputs of a speckle filter.

Similarly to the case of target vectors filtering, a incoherent covariance matrix may be built from the APP coherent vector, $\underline{E}_{PPi}$, as

$$[C_2]_{PPi} = \underline{E}_{PPi} E_{PPi}^{\dagger} \tag{14}$$

The express of the boxcar and Lee filters are similar to those introduced for the (3×3) $[C]$ and $[T]$ matrices.

6.2.2.1 APP and APG boxcar filter

Filtered estimates, $[\tilde{C}_2]_{PPi}$, are constructed by computing the sample mean over each pixel neighborhood, defined by a sliding window of $(N_w \times N_w)$ pixels

$$[\tilde{C}_2]_{PPi} = \left\langle [C_2]_{PPi} \right\rangle_{N_w} \tag{15}$$

6.2.2.2 APP and APG Lee filter

The adaptation of Lee MMSE filter to APP and APG data formats leads to the following intensity vector estimate expression

$$[\tilde{C}_2]_{PPi} = \left\langle [C_2]_{PPi} \right\rangle_{N_w} + k\left([C_2]_{PPi} - \left\langle [C_2]_{PPi} \right\rangle_{N_w} \right) \tag{16}$$

where k is an adaptive filtering coefficient, based on local statistics, given by

$$k = \frac{\mathrm{var}\langle span \rangle - E^2(\langle span \rangle)\sigma_n^2}{\mathrm{var}\langle span \rangle[1 + \sigma_n^2]}$$

with $\sigma_n^2 = \dfrac{1}{L}$ the a priori speckle variance
$$\tag{17}$$

where *span* represents the trace of the covariance matrix.

6.3 Incoherent decomposition

Averaged $[C_2]$ matrices do not correspond to an PP scattering mechanism and may the be decomposed in order to extract a pure PP representation.

A $[C_2]$ may be decomposed on its eigenvector basis as follows

$$[C_2] = [V_2][\Lambda_2][V_2]^{\dagger} = \lambda_1 v_1 v_1^{\dagger} + \lambda_2 v_2 v_2^{\dagger} \tag{18}$$

where $[\Lambda_2]$ and $[V_2]$ represent (2×2) real eigenvalue and special unitary eigenvector matrices respectively.

Each unitary eigenvector, v_i, may be parameterized using 3 real parameters

$$v_i = e^{j\xi} [\cos\alpha_i \ \sin\alpha_i e^{j\delta_i}]^T \tag{19}$$

The eigenvalue set may also be parameterized using two real variables

$$\lambda_1 + \lambda_2 = Tr[C_2] = span \ \text{ and } \ H = -P1\log_2(P1) - P2\log_2(P_2) \tag{20}$$

$$\text{with } P_i = \frac{\lambda_i}{\lambda_1 + \lambda_2}$$

where H represents the scattering mechanism entropy while P_i is the pseudo-probability of one of the decomposed orthogonal scattering mechanisms, by convention $P_1 \geq P_2$.

The anisotropy, A, may also be used to characterize the scattering phenomenon

$$A = \frac{P_1 - P_2}{P_1 + P_2} \tag{21}$$

One may note that the joint use of H and A is redundant since the characterization of a two probability set requires only a single real parameter.

The physical signification of H and A is identical to the one derived for the decomposition of (3×3) [C] and [T] matrices.

Similarly to the (3×3) case, an average scattering mechanism may be rebuilt from the pseudo-probability set as

$$(\underline{\alpha}, \underline{\delta}) = P_1(\alpha_1, \delta_1) + P_2(\alpha_2, \delta_2) \tag{22}$$

On the opposite, the interpretation of $(\underline{\alpha}, \underline{\delta})$ is totally different from one PP configuration to the other.

6.4   Supervised segmentation

It has been demonstrated, in the chapter dedicated to polarimetric SAR data classification, that statistical supervised classification of data requires the derivation of data ML statistics.

6.4.1   APS format data segmentation

6.4.1.1   Segmentation of coherent data

APS format consist of complex two-element vector composed of coherent scattering coefficients and may be represented, in a general way, as follows

$$\underline{E} = \begin{bmatrix} S_{pq} \\ S_{rs} \end{bmatrix} \tag{23}$$

where the subscripts p, q, r, s represent polarization channels.

It has been verified that when the radar illuminates an area of random surface of many elementary scatterers, $\underline{E}$ can be modeled as having a multivariate complex circular gaussian probability density function $N_C(0, [\Sigma])$ of the form

$$P(\underline{k}) = \frac{\exp(-\underline{E}^\dagger [\Sigma]^{-1} \underline{E})}{\pi^q \|[\Sigma]\|} \tag{24}$$

where q stands for the number of elements of $\underline{E}$, equal to three in the monostatic case, $||$ represents the determinant, and $[\Sigma] = E(\underline{E}\underline{E}^{\dagger})$ is the global (3×3) covariance matrix of $\underline{E}$.

The likelihood of a target vector $\underline{E}$ given a cluster Θ_i is then given by

$$P\big(\underline{E}|\Theta_i\big) = \frac{\exp(-\underline{E}^{\dagger}[\Sigma_i]^{-1}\underline{E})}{\pi^q |[\Sigma_i]|} \tag{25}$$

With $[\Sigma_i]$ the covariance matrix of cluster Θ_i computed during the learning phase. In practice the actual value of $[\Sigma_i]$ remains unknown and the covariance matrix is replaced by its maximum likelihood estimate $[\hat{\Sigma}_i]$ defined as

$$[\hat{\Sigma}_i] = \frac{1}{n_i} \sum_{p \in \Theta_i} \underline{E}\underline{E}^{\dagger} \tag{26}$$

With n_i the number of pixels belonging to the training cluster Θ_i.

From a computational point of view, it is generally preferable to deal with log-likelihood function, $L\big(\underline{E}|\Theta_i\big)$ instead of the expression given in **Erreur ! Source du renvoi introuvable.**

$$L\big(\underline{E}|\Theta_i\big) = -\ln\big|[\hat{\Sigma}_i]\big| - tr\big([\hat{\Sigma}_i]^{-1}\underline{E}\underline{E}^{\dagger}\big) - q\ln\pi \tag{27}$$

The logarithm function being strictly increasing with its argument, the optimal decision rule is transformed, in the case of $\underline{E}$ vector segmentation, to :

$$\text{Decide } p \in \Theta_i \text{ if } \Theta_i = Arg \min d\big(\underline{E}|[\hat{\Sigma}_i]\big) \text{ with}$$
$$d\big(\underline{E}|[\hat{\Sigma}_i]\big) = +\ln\big|[\hat{\Sigma}_i]\big| + tr\big([\hat{\Sigma}_i]^{-1}\underline{E}\underline{E}^{\dagger}\big) \tag{28}$$

Where the variable $d\big(\underline{E}|[\hat{\Sigma}_i]\big)$ may be assimilated to a statistical distance, derived from the opposite of log-likelihood function without constant terms.

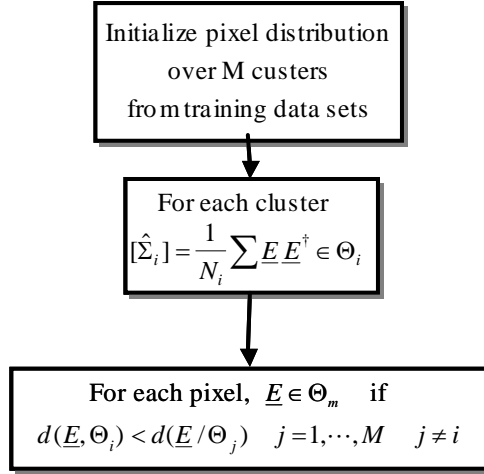The supervised segmentation algorithm may be summarized as follows

Figure 1 Supervised ML segmentation scheme

6.4.1.2   Segmentation of incoherent data

It has been shown that assuming that $\underline{E}$ vectors have a $N_C(0,[\Sigma])$ distribution, a sample n-look covariance matrix $[C_2] = 1/n\sum_n \underline{E}\,\underline{E}^\dagger$, follows a complex Wishart distribution with n degrees of freedom, $W_C(n,[\Sigma])$, given by

$$p([C_2]) = \frac{n^{qn}\big|[C_2]\big|^{n-q}\exp(-tr(n[\Sigma]^{-1}[C_2]))}{K(n,q)\big|[\Sigma]\big|^{n}}$$

$$\text{with}\quad K(n,q) = \pi^{q(q-1)/2}\prod_{i=1}^{q}\Gamma(n-i+1)$$

(29)

Where $\Gamma(.)$ represents the Gamma function.

A development similar to the one presented in the case of coherent vectors leads to the following decision rule

$$\text{Decide } p\in\Theta_i \text{ if } \Theta_i = Arg\max L\big([C_2]\big|[\hat{\Sigma}_i]\big) \text{ with}$$

$$L\big([C_2]\big|[\hat{\Sigma}_i]\big) = -n\ln\big|[\hat{\Sigma}_i]\big| - ntr\big([\hat{\Sigma}_i]^{-1}[C_2]\big) + qn\ln n + (n-q)\ln\big|[C_2]\big| - \ln K(n,q)$$

(30)

With $[\hat{\Sigma}_i]$, the maximum likelihood estimate of the coherency matrix defined as

$$[\hat{\Sigma}_i] = \frac{1}{n_i}\sum_{p\in\Theta_i}[C_2]$$

(31)

Taking the opposite of the lower expression of relation (30) and removing terms that do not depend on the cluster under test, the optimal decision rule becomes :

$$\text{Decide } p\in\Theta_i \text{ if } \Theta_i = Arg\min d\big([C_2]\big|[\hat{\Sigma}_i]\big) \text{ with}$$

$$d([C_2]\big|[\hat{\Sigma}_i]\big] = +\ln\big|[\hat{\Sigma}_i]\big| + tr\big([\hat{\Sigma}_i]^{-1}[C_2]\big)$$

(32)

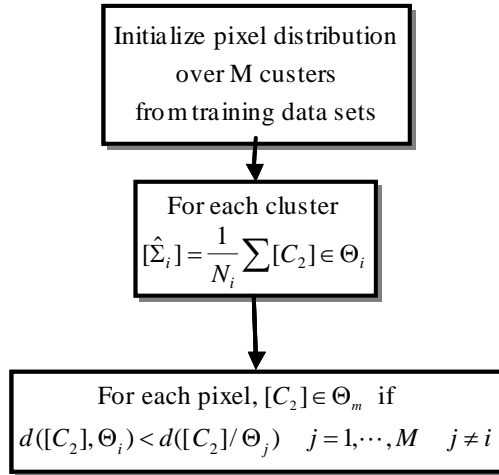The supervised segmentation algorithm may be summarized as follows

Figure 2 Supervised ML segmentation scheme

6.4.2   APP and APG data segmentation

The segmentation presented is simplified by assuming that APP and APG intensities are independent. In this case, their joint probability is given by

$$p([C_{I2}]) = \frac{n^{qn}\big|[C_{I2}]\big|^{n-q} \exp(-tr(n[\Sigma_I]^{-1}[C_{I2}]))}{K(n,q)\big|[\Sigma_I]\big|^{n}}$$

(33)

$$\text{with}\quad K(n,q) = \pi^{q(q-1)/2}\prod_{i=1}^{q}\Gamma(n-i+1)$$

Where $[C_{I2}]$ and $[\Sigma_I]$ are (2×2) real matrices with off-diagonal elements equal to zero and built as follows

$$[C_{I2}] = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix} \text{ and } [\Sigma_I] = \begin{bmatrix} E(I_1) & 0 \\ 0 & E(I_2) \end{bmatrix}$$

(34)

The optimal decision rule for APP and APG format data is then given by

$$\text{Decide } p \in \Theta_i \text{ if } \Theta_i = Arg\min d\big([C_{I2}]\big|[\hat{\Sigma}_{Ii}]\big) \text{ with}$$
$$d([C_{I2}]\big|[\hat{\Sigma}_{Ii}] = +\ln\big|[\hat{\Sigma}_{Ii}]\big| + tr\big([\hat{\Sigma}_{Ii}]^{-1}[C_{I2}]\big)$$

(35)