3. SINGLE VS MULTI-POLARIZATION DESCRIPTORS

3.1 Polarisation and Surface Scattering

The main problem for the quantitative estimation of soil moisture and/or surface roughness from SAR data lies in the separation of their individual effects on the backscattered signal. Polarimetry plays here an important role as it allows either a direct separation or a parameterisation of roughness and moisture effects within the scattering problem. The scattering problem of electromagnetic waves from randomly rough surfaces has been an actual research topic over decades and is still not satisfactorily solved due to the lack of an exact closed-form solution. However, for many practical application, approximate solutions are sufficient. In the absence of any direct relationship between surface parameters and backscattering signal models empirical relations have been developed. In the field of radar remote sensing the most common approximation methods are based on the evaluation of backscattering amplitudes considering single or dual-channel SAR data. The choice of an appropriate scattering model is essential for the quantitative estimation of the surface parameters. On the one hand it must contain enough physical structure while on the other hand it should have a right balance between the amount of parameters needed for their description and available observables. As natural surfaces are complex stochastic objects *a priori* information and assumptions can be used to simplify the inversion problem. Hence, for some of the scattering model in order to obtain an accurate estimate of soil moisture, information about the surface roughness was required or roughness has been considered as a disturbing effect and several conditions have been developed in order to minimise its influence (Hajnsek, 2001).

However, an independent estimate of roughness conditions is not possible by using only a single polarisation and single- frequency SAR system. Increasing the amount of obervables by using a fully polarimetric data set, the amount of surface parameters and its estimation quantity increases. However, the main limitation of using models based on polarimetric backscattering amplitudes is their insufficiency to deal - at leas in a practical way - with diffuse or secondary scattering processes, depolarisation effects caused by the surface roughness, and the presence of multiplicative and / or additive noise components.

A large class of natural surface scatterers, is characterised by secondary and/or multiple scattering effects. With increasing surface roughness, relative to the wavelength, the effect of multiple scattering becomes stronger, generating an adequate $|HV|$ scattering component. Dihedral scattering due to small correlation lengths characterised by $|HH| > |VV|$, and/or diffuse scattering ($|HV|$ contribution) affects the backscattered signal. Also, effects induced by the presence of vegetation cover can not be accounted with surface scattering models. Both effects lead to a violation of the required conditions of most models and/or to biased estimation of the roughness and moisture parameters.

3.1.1 Polarimetric Coherence

The first way followed in order to extend the observation space was by incorporating polarimetric phase information in form of (second-order) correlation (coherence) coefficients between different polarisations. Indeed, correlation coefficients at different polarisations have been reported to be sensitive to surface parameters. The *HH-VV* correlation coefficient

$$\gamma_{HHVV} := \frac{|< S_{HH}\ S_{VV}^{*} >|}{\sqrt{< S_{HH}\ S_{HH}^{*} > < S_{VV}\ S_{VV}^{*} >}} \tag{1}$$

has been evaluated with respect to its sensitivity to dielectric constant and/or rms height condition of rough surfaces in (Borgeaud, 1994). An increased correlation to soil moisture conditions within a certain roughness range has been reported. In contrary, (*HH*+*VV*)-(*HH*-*VV*) correlation coefficient

$$\gamma_{(HH+VV)(HH-VV)} := \frac{|< \left(S_{HH} + S_{VV}\right)\ \left(S_{HH} - S_{VV}\right)^{*} >|}{\sqrt{< \left|S_{HH} + S_{VV}\right|^{2} > < \left|S_{HH} - S_{VV}\right|^{2} >}} \tag{2}$$

has been found to correlate with surface roughness, independent on the surface moisture content and the local incidence angle (Hajnsek, 2001).

Further investigations in (Mattia, 1997) demonstrated that the circular polarisation correlation coefficient

$$\gamma_{LLRR} := \frac{|< S_{LL}\ S_{RR}^{*} >|}{\sqrt{< S_{LL}\ S_{LL}^{*} > < S_{RR}\ S_{RR}^{*} >}} \tag{3}$$

is widely independent on the dielectric properties of rough surfaces and depends only on the surface roughness. Its independence from the azimuth topographic tilt has been reported in (Schuler 2002). Thus, γ_{LLRR} can be used for robust quantitative surface roughness estimates.

More recently the polarimetric scattering anisotropy obtained from the eigen-values of the polarimetric coherence matrix has been evaluated to be a sensitive estimate for the surface roughness (Cloude 1999). This is in accordance with the observations about the circular polarisation correlation coefficient as the anisotropy A can be interpreted as a generalised rotation invariant expression of γ_{LLRR}

in case where the coherency matrix is diagonal.

$$A := \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3} \quad \lambda_2,\ \lambda_3 \text{ Eigenvalues of [T]} \tag{4}$$

Both parameters, γ_{LLRR} and A, are by definition normalised between 0 and 1 and have – in first order - a direct linear relationship to the surface roughness

$$1 - \gamma_{LLRR} = ks = 1 - A \quad \text{for} \qquad 0 \le ks \le 1 \tag{5}$$

The price to be payed for using the rotation invariant anisotropy is its very ‘noisy’ nature especially at the low range of its range i.e. for smooth surfaces (Hajnsek 2002). In this case the amplitudes of the secondary scattering mechanisms, expressed by the second and third eigenvalues are very small, down to -25 [dB] or even less. As this is close to the system noise floor, A is strongly affected by additive noise effects. However, by additive filtering the influence can be reduced.

Concerning the valuable surface roughness range, radar sensors operating at lower frequencies (L-band) allow the coverage of a sufficiently wide range of natural surfaces, as it can be seen in Tab 1.

Band ID	**Wavelength [cm]**	***ks*** **for *rms height* 0.5 [cm]**	***ks*** **for *rms height* 4 [cm]**
X	*3*	*1.04*	*8.37*
C	*5*	*0.52*	*4.19*
S	*10*	*0.31*	*2.51*
L	*23*	*0.14*	*1*
P	*68*	*0.05*	*0.37*

Tab.1: The surface roughness range for natural surface calculated for different wavelength

Based on these experimental observations a polarimetric scattering model using as a starting point the SPM (Bragg-Model) an extended Bragg model (X-Bragg), has been developed, which allows quantitative estimation of roughness and dielectric constant over a wide range of natural bare surfaces. X-Bragg assumes reflection symmetric surfaces, where the axis of symmetry is defined by the mean normal to the surface vector. It accounts for cross-polarised as well as depolarisation effects. The application of the model to experimental data demonstrate a high inversion accuracy shows a good agreement between inverted values and ground measurements (Hajnsek, 2001).

However, the main limitation for surface parameters estimation from polarimetric SAR data is the presence of vegetation. This combined with the fact that most natural surfaces are temporarily or permanently covered by vegetation restricts significantly the importance of radar remote sensing for a wide spectrum of geophysical and environmental applications. However, the evolution of radar technology and techniques allows optimism concerning the vitiation of this limitation.

3.2 The Extended or X-Bragg model

Based on these experimental observations, a new model for the investigation of surface parameters from polarimetric SAR data is introduced in this paper. The proposed model is a two component model including a Bragg scattering term and a roughness induced rotation symmetric disturbance. In order to decouple the real part of the dielectric constant ε' from surface roughness s, the model is addressed in terms of the polarimetric scattering entropy (H), scattering anisotropy (A) and alpha angle (α) which are derived from the eigenvalues and eigenvectors of the polarimetric coherency matrix (Cloude et al. 1999). This allows the implementation of a simple inversion algorithm for both surface roughness and soil moisture.

3.2.1 Second Order Scattering Description of Surface Scatterers

In order to describe correlation properties of natural scatterers, the polarimetric coherency matrix which contains the second order moments of the scattering process is introduced. Its formation is based on the introduction of a scattering vector $\vec{k}_P$ as the vectorisation of the scattering matrix [S] using the Pauli spin matrices basis set (Cloude et al. 1996, Cloude et al. 2001).

$$[S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \rightarrow \vec{k}_P := \frac{1}{\sqrt{2}} \left[S_{HH} + S_{VV} \quad S_{HH} - S_{VV} \quad 2\, S_{HV} \right]^T \tag{6}$$

The coherency matrix [T] is formed by averaging the outer product of $\vec{k}_P$ as

$$[T] := \left\langle \vec{k}_P \cdot \vec{k}_P^+ \right\rangle = \frac{1}{2} \begin{bmatrix} \left\langle |S_{HH} + S_{VV}|^2 \right\rangle & \left\langle (S_{HH} + S_{VV})(S_{HH} - S_{VV})^* \right\rangle & 2\left\langle (S_{HH} + S_{VV}) S_{HV}^* \right\rangle \\ \left\langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \right\rangle & \left\langle |S_{HH} - S_{VV}|^2 \right\rangle & 2\left\langle (S_{HH} - S_{VV}) S^*{}_{HV} \right\rangle \\ 2\left\langle S_{HV}(S_{HH} + S_{VV})^* \right\rangle & 2\left\langle S_{HV}(S_{HH} - S_{VV})^* \right\rangle & 4\left\langle |S_{HV}|^2 \right\rangle \end{bmatrix} \tag{7}$$

The diagonal elements of [T] are given by the real backscattered power values while the off-diagonal elements contain the complex cross correlations between the elements of $\vec{k}_P$. [T] is by definition hermitian positive semi-definite, which implies that it has real non-negative eigenvalues and orthogonal eigenvectors, and can always be diagonalised by an unitary similarity transformation of the form (Cloude et al. 1997)

$$[T] = [U_3][\Lambda][U_3]^{-1} \text{ where } [\Lambda] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}, \quad [U_3] = \left[\vec{e}_1, \vec{e}_2, \vec{e}_3 \right]^T \tag{8}$$

[Λ] is a diagonal matrix with elements the real non-negative eigenvalues, $0 \leq \lambda_1 \leq \lambda_2 \leq \lambda_3$ of [T], and [U_3] is the unitary eigenvector matrix with columns corresponding to the orthonormal eigenvectors $\vec{e}_1$, $\vec{e}_2$ and $\vec{e}_3$. The diagonalisation of the coherency matrix [T] may be interpreted as its decomposition into a non-coherent sum of three independent scattering contributions

$$[T] = [U_3][\Lambda][U_3]^{-1} = \lambda_1 \left(\vec{e}_1 \cdot \vec{e}_1^+ \right) + \lambda_2 \left(\vec{e}_2 \cdot \vec{e}_2^+ \right) + \lambda_3 \left(\vec{e}_3 \cdot \vec{e}_3^+ \right) \tag{9}$$

The contribution of each scattering mechanism in terms of power is given by the appropriate eigenvalue. The information about which 'kind' of scattering mechanism is obtained is contained in the corresponding eigenvectors.

There are three important physical features arising directly from the diagonalisation of the coherency matrix. The first two are obtained from the eigenvalues of [T] which - normalised by the absolute scattering magnitudes - can be interpreted as scattering probabilities p_i (Cloude et al. 2001)

$$p_i := \frac{\lambda_i}{\sum_{j=1}^{3} \lambda_j} = \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3} \quad \rightarrow \quad p_1 + p_2 + p_3 = 1 \tag{10}$$

Using the probabilities p_i two ratios for the description of an arbitrary rough surface can be defined: the polarimetric scattering entropy H and the scattering anisotropy A, which are defined as

$$H = -\sum_{i=1}^{3} p_i \log_3 p_i \tag{11}$$

$$A = \frac{p_2 - p_3}{p_2 + p_3} \tag{12}$$

Both parameters vary between 0 and 1. The entropy can be interpreted as a measure of the randomness of the scattering process. The anisotropy defines the relation between the second and the third eigenvalues, and is a measure for the difference of the secondary scattering mechanisms. For smooth surfaces, H becomes zero implying a non-depolarising scattering process described by a single scattering matrix and increases with surface roughness. Depolarising surfaces are characterised by non-zero entropy values. A can be zero even for rough surfaces. For surfaces characterised by intermediate entropy values, a high anisotropy indicates the presence of only one strong secondary scattering process, while a low anisotropy indicates the appearance of two equally strong scattering processes. For azimuthaly symmetric surfaces $\lambda_2=\lambda_3$ by definition and A becomes 0 (Cloude et al. 1996, Cloude et al. 2001). In this sense, the anisotropy provides complementary information to the entropy and facilitates the interpretation of the surface scatterer. The great advantage of these two parameters arises from the invariance of the eigenvalue problem under unitary transformations: The same surface leads to the same eigenvalues and consequently to the same entropy and anisotropy values independently on the basis used to measure the corresponding scattering matrix.

The third important parameter is obtained from the eigenvectors of [T]. Each eigenvector $\vec{e}_i$ can be expressed in terms of five angles as (Cloude et al. 1997)

$$\vec{e}_i = \left[\cos\alpha_i \exp\left(i\phi_{1i}\right) \quad \sin\alpha_i \cos\beta_i \exp\left(i\phi_{2i}\right) \quad \sin\alpha_i \sin\beta_i \exp\left(i\phi_{3i}\right)\right]^T \tag{13}$$

The β_i angles can be interpreted as a rotation of the corresponding eigenvector $\vec{e}_i$ in the plane perpendicular to the scattering plane, while ϕ_{1i}, ϕ_{2i} and ϕ_{3i} are accounting for the phase relations between the elements of $\vec{e}_i$. More important in the context of this work is the mean scattering angle α defined as

$$\alpha = p_1\alpha_1 + p_2\alpha_2 + p_3\alpha_3 \tag{14}$$

The alpha angle α indicates the "type" of the mean scattering process occurring (Cloude et al. 1999). Its interpretation in terms of the surface scattering problem will be given in the following sections.

Returning now to the SPM, the corresponding scattering vector $\vec{k}_P$ is given by

$$\vec{k}_P = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} + S_{VV} \\ S_{HH} - S_{VV} \\ 2S_{HV} \end{bmatrix} = m_s \begin{bmatrix} R_s + R_P \\ R_S - R_P \\ 0 \end{bmatrix} = m \begin{bmatrix} \cos\alpha \exp(i\phi_1) \\ \sin\alpha \exp(i\phi_2) \\ 0 \end{bmatrix} \tag{15}$$

where m denotes the absolute scattering amplitude. The coherency matrix of a Bragg scatterer follows as

$$[T] = \left\langle \vec{k}_P \vec{k}_P^{+} \right\rangle = m_s^2 \begin{bmatrix} \left\langle |R_S + R_P|^2 \right\rangle & \left\langle (R_S + R_p)(R_S - R_P)^* \right\rangle & 0 \\ \left\langle (R_S - R_P)(R_S + R_P)^* \right\rangle & \left\langle |R_S - R_P|^2 \right\rangle & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{16}$$

Two of the three off-diagonal elements disappear as a consequence of zero cross-polarisation, while the third one depends only on the surface dielectric constant and the radar incidence angle. The corresponding normalised correlation coefficient is one

$$\gamma_{(HH+VV)(HH-VV)} := \frac{|\left\langle (S_{HH} + S_{VV})(S_{HH} - S_{VV})^* \right\rangle|}{\sqrt{\left\langle | S_{HH} + S_{VV} |^2 \right\rangle \left\langle | S_{HH} - S_{VV} |^2 \right\rangle}} = \frac{|\left\langle (R_S + R_p)(R_S - R_P)^* \right\rangle|}{\sqrt{\left\langle | R_S + R_P |^2 \right\rangle \left\langle | R_S - R_P |^2 \right\rangle}} = 1 \tag{17}$$

as a consequence of the inability of the SPM to describe depolarisation effects.

Regarding now the interpretation of the alpha angle, from (15) follows that α – similar to the co-polarised ratio R_s / R_p – is independent of roughness, and therefore, can be used for the estimation of the dielectric constant if the local incidence angle is known

$$\alpha = \arccos\left(\frac{|R_S + R_P|}{\sqrt{|R_S + R_P|^2 + |R_S - R_P|^2}} \right) \tag{18}$$

As shown in **Figure 1**, for a dry surface the alpha angle α is small and increases with increasing soil moisture. The highest sensitivity occurs between 0 and 20 m_v [vol. %] and saturates with further increasing of m_v.

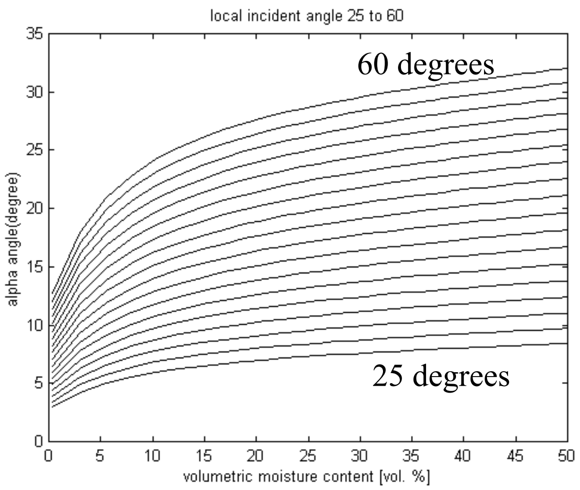

Figure 1 The relation of the alpha angle to the volumetric soil moisture

Summarising, the main limitations of the SPM are its small surface roughness validity range – below 0.3 *ks* and the saturation of its sensitivity to soil moisture content above m_v 20 [vol. %]. Its inability to describe *depolarisation* effects restricts its usefulness regarding the interpretation and inversion of experimental data from natural surfaces. However, the robustness of SPM inside its validity range and its relevant physical background leaded to several investigations to use it as a valuable starting point for an extended model.

As mentioned in the previous section, two main effects have to be introduced in order to extend the Bragg scattering model for a wider range of roughness conditions: non-zero *cross-polarised* backscattering and *depolarisation*. In fact, conventional two- or multiple-scale extensions introduce cross-polarisation but fail to express depolarisation effects. An alternative way to introduce roughness disturbance is to model the surface as a reflection symmetric depolariser by rotating the Bragg coherency matrix [*T*] about an angle *ß* in the plane perpendicular to the scattering plane (Lee et al. 2000) as

$$[T(\beta)]=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos 2\beta & \sin 2\beta\\ 0 & -\sin 2\beta & \cos 2\beta\end{bmatrix}\begin{bmatrix}\langle |R_S+R_P|^2\rangle & \langle (R_S-R_P)(R_S+R_P)^*\rangle & 0\\ \langle (R_S+R_P)(R_S-R_P)^*\rangle & \langle |R_S-R_P|^2\rangle & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos 2\beta & -\sin 2\beta\\ 0 & \sin 2\beta & \cos 2\beta\end{bmatrix} \quad (19)$$

and performing a configurational averaging over a given distribution *P*(*ß*) of *ß:*

$$[T]=\int_0^{2\pi}[T(\beta)]P(\beta)d\beta \quad (20)$$

The width of the assumed distribution corresponds to the amount of roughness disturbance of the modelled surface. Assuming *P*(*ß*) to be a uniform distribution about zero with width β_1 (see **Figure 2**)

$$P(\beta)=\begin{cases}\dfrac{1}{2\beta_1} & |\beta|\leq\beta_1\\ & 0\leq\beta_1\leq\dfrac{\pi}{2}\end{cases} \quad (21)$$

the coherency matrix for the rough surface becomes

$$[T]=\begin{bmatrix}T_{11} & T_{12} & T_{13}\\ T_{21} & T_{22} & T_{23}\\ T_{31} & T_{32} & T_{33}\end{bmatrix}=\begin{bmatrix}C_1 & C_2\sin c(2\beta_1) & 0\\ C_2\sin c(2\beta_1) & C_3(1+\sin c(4\beta_1)) & 0\\ 0 & 0 & C_3(1-\sin c(4\beta_1))\end{bmatrix} \quad (22)$$

(11.50)

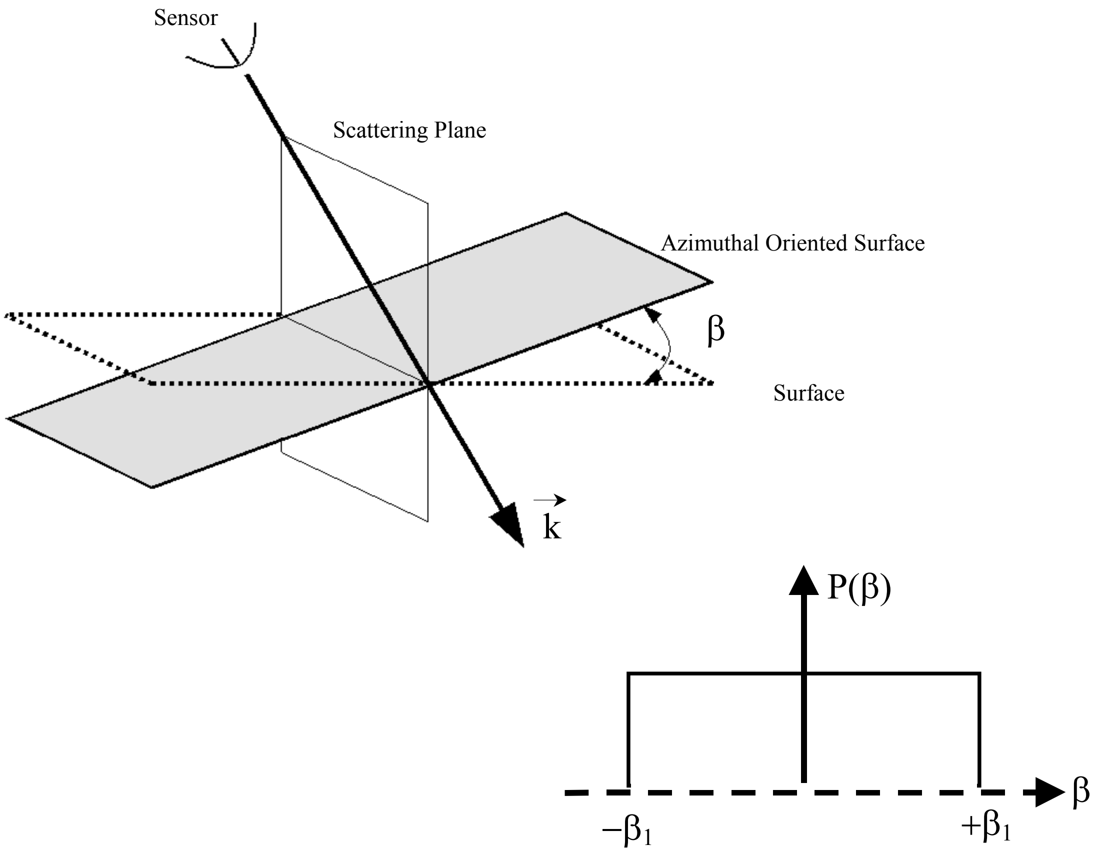

Figure 2 Schematic representation of the uniform distribution of the slope

with sinc(x) = sin(x) / x. The coefficients C_1, C_2 , and C_3 describing the Bragg components of the surface, and are given by

$$C_1 = |R_S + R_P|^2 \qquad C_2 = (R_S + R_P)(R_S^* - R_P^*) \qquad C_3 = \frac{1}{2}|R_S - R_P|^2 \tag{23}$$

Equation (22) represents the coherency matrix of an extended Bragg surface characterised by cross-polarised energy and at the same time a polarimetric coherence less than one

$$\gamma_{(HH+VV)(HH-VV)} := \frac{T_{12}}{\sqrt{T_{11}\,T_{22}}} = \frac{\sin c(2\beta_1)}{\sqrt{(1+\sin c(4\beta_1))/2.}} \leq 1 \tag{24}$$

The width of the distribution β_1 describes the roughness component and controls both, the level of cross-polarised power and the *(HH+VV)(HH-VV)* coherence. Fig. (11.14) show the variation of *(HH+VV)(HH-VV)* coherence (Fig. (11.14)) and normalised cross-polarised power according to (22). In the limit of a smooth surface (i.e. $\beta_1 = 0$), the *(HH+VV)(HH-VV)* coherence is one and the *HV* backscattered power zero. In this case, the coherency matrix obtains the form of the "pure" Bragg coherency matrix (see (16)). With increasing roughness (i.e. increasing β_1), the *HV* power increases, while the *(HH+VV)(HH-VV)* coherence decreases monotonically from 1 to zero. In this high roughness limit, the surface becomes azimuthally symmetric.

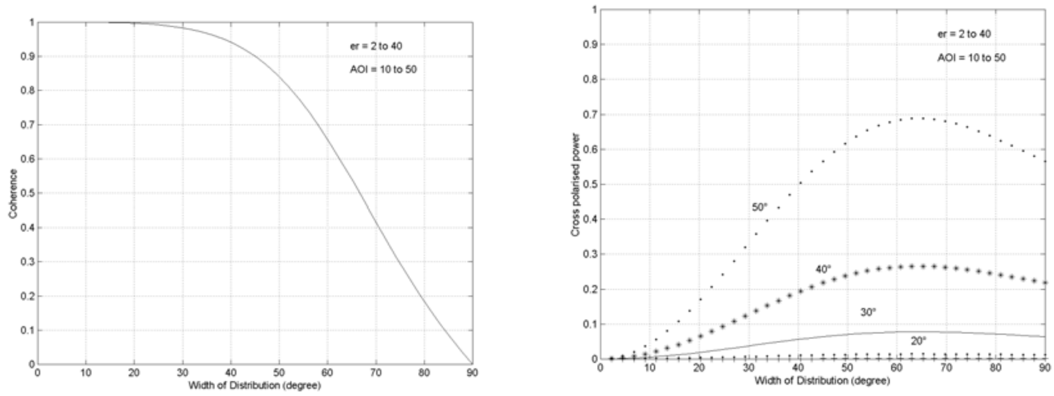

Figure 3 Variation of the coherence and the cross polarised power with increasing β1

Furthermore, it is important to realise that, while the increase of cross polarised power with roughness depends on the dielectric constant of the surface (and the incidence angle) the decrease of the (*HH+VV)*(*HH-VV)* coherence is independent from both.

On the other hand, according to (22), the ratio

$$\frac{T_{22}+T_{33}}{T_{11}}=\frac{\left|R_S-R_P\right|^2}{\left|R_S+R_P\right|^2} \tag{25}$$

is independent on the surface roughness and depends only on the dielectric constant of the surface and the incidence angle as shown in **Figure 4**. Thus, the extended Bragg model is characterised by an inherent separation of roughness and moisture effects obtained in form of ratios of the elements of the coherency matrix allowing an independent estimation of these parameters.

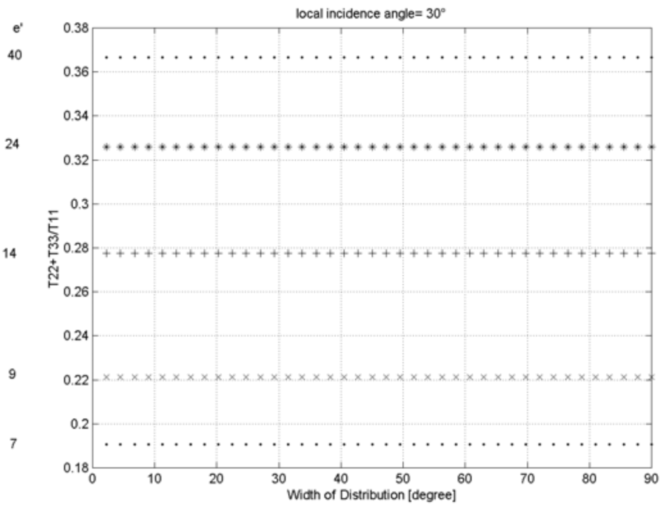

Figure 4 . The polarisation ratio as a function of the β1 parameter.