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( $19^{\text {th }}$ April 2004. $3 \sim 6 \mathrm{pm}$. You may answer the questions in Korean or English.)

| 학과 |  | 학번 |  | 성명 |  |
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All questions are related with each other in sequence. You may find it easier to answer the questions in order.

1. A SAR antenna transmits the following chirp signal.

$$
s(t)=\exp \left[j 2 \pi\left(f_{c} t+K t^{2} / 2\right)\right], \quad|t|<\tau_{p} / 2
$$

(a) Define and draw the phase $\phi(t)$ of the signal. (3 point)
(b) Find and draw the frequency $f(t)$ of the signal. (5 point)
(c) Find the chirp bandwidth $B_{R}$ of the signal. (2 point)
2. The returned signal from a point target is again a delayed chirp signal, $v(t)=s\left(t-t_{n}\right)$, where $t_{n}=2 R / c$ is the 2-way travel time of the pulse from the sensor to the target. The range compression is performed by the following matched filtering:

$$
g(t)=\int_{-\infty}^{\infty} s^{*}\left(t^{\prime}-t\right) v\left(t^{\prime}\right) d t^{\prime}
$$

To evaluate this integration, you need to determine the effective range of integration carefully.

(a) Determine the effective range of integration by filling the empty boxes in the figure above. (5 point)
(b) Prove that the matched filtering results in

$$
\begin{equation*}
g(t)=\exp \left[j 2 \pi f_{c}\left(t-t_{n}\right)\right] \frac{\sin \left[\pi K\left(t-t_{n}\right)\left(\tau_{p}-\left|t-t_{n}\right|\right)\right]}{\pi K\left(t-t_{n}\right)} \tag{20point}
\end{equation*}
$$

(c) This function can be approximated as:

$$
g(t) \approx \exp \left(j \omega_{c} t\right) \exp (-j 4 \pi R / \lambda) \frac{\sin \left[\pi K \tau_{p}\left(t-t_{n}\right)\right]}{\pi K\left(t-t_{n}\right)}
$$

Find and draw the amplitude $|g(t)|$ in term of $t$ centered at $t_{n}$, specifying the maximum amplitude and several times that the amplitude fall to zero. (5 point)
(d) The half-width of the main lobe can be used as a definition of range time resolution (originally 3 dB width of the main lobe). Express the range time resolution in terms of $K$ and $\tau_{p}$, and again, in terms of chirp bandwidth $B_{R}$. (5 point)
(e) Find the range compression ratio which is defined by the chirp duration divided by the range time resolution. (5 point)
3. As the satellite moves forward, many pulses are to be transmitted and backscattered from the point target. Each returned pulse is then range-compressed and sampled at $t=t_{n}$. The carrier structure of the signal is stripped away by the linear operation of complex demodulation. The range-compressed signal of each pulse transmitted at slow time $s$, is then (see the figure below)

$$
\hat{g}(s)=\exp [j \phi(s)]=\exp [-j 4 \pi R(s) / \lambda]
$$

The Taylor's series expansion of $R(s)$ centered at $s=S_{c}$ (the time at which the target is located at the beam center) is:

$$
R(s)=R_{c}+\dot{R}\left(s_{c}\right)\left(s-s_{c}\right)+\ddot{R}\left(s_{c}\right)\left(s-s_{c}\right)^{2} / 2+\cdots \cdots .
$$

(a) Using the definition of Doppler frequency:

$$
\begin{aligned}
& f_{D}(s)=\dot{\phi}(s) / 2 \pi \\
& \dot{f}_{D}(s)=\ddot{\phi}(s) / 2 \pi
\end{aligned}
$$

show that

$$
\hat{g}(s)=\exp \left(-j 4 \pi R_{c} / \lambda\right) \exp \left\{j 2 \pi\left[f_{D c}\left(s-s_{c}\right)+f_{R}\left(s-s_{c}\right)^{2} / 2\right]\right\},\left|s-s_{c}\right|<S / 2
$$

Here $S$ is the coherent integration time, which means the duration the target is within the view of radar pulse. $f_{D c}=f_{D}\left(s_{c}\right)$ is Doppler centroid, and $f_{R}=\dot{f}_{D}\left(s_{c}\right)$ is Doppler rate.
(b) The above signal can be interpreted as the returned pulse of the linear chirp pulse:

$$
h(s)=\exp \left[j 2 \pi\left(f_{D c} s+f_{R} s^{2} / 2\right)\right],|s|<S / 2
$$

Therefore, we can compress the signal with respect to $s$ using matched filtering as:

$$
\varsigma(s)=\int_{-\infty}^{\infty} h^{*}\left(s^{\prime}-s\right) \hat{g}\left(s^{\prime}\right) d s^{\prime}
$$

Prove that the above integration results in

$$
\varsigma(s) \approx \exp \left(-j 4 \pi R_{c} / \lambda\right) \exp \left[j 2 \pi f_{D_{c}}\left(s-s_{c}\right)\right] \frac{\sin \left[\pi f_{R} S\left(s-s_{c}\right)\right]}{\pi f_{R}\left(s-s_{c}\right)} \cdot \text { (20 point) }
$$

(c) Define azimuth time resolution ( $\delta \mathrm{s}$ ) and azimuth spatial resolution ( $\delta x$ ) in terms of $S$ and $f_{R}$, and again, in terms of Doppler bandwidth $B_{D}$. (5 point)
(d) Using the fact that the nominal beam width of antenna length $L_{a}$ is $\theta_{H}=\lambda / L_{a}$, derive

$$
\text { that } S=\lambda R_{c} / V_{s} L_{a} .(5 \text { point })
$$

(e) Using $R(s) \approx R_{c}-V_{s} \sin \theta_{s}\left(s-s_{c}\right)+V_{s}^{2}\left(s-s_{c}\right)^{2} / 2 R_{c}$, find $f_{D c}$ and $f_{R}$. (5 point)
(f) Prove that the azimuth resolution is $\delta x=L_{a} / 2$. (5 point)
4. What does SAR stand for? (2 point)
5. Describe what you have learnt so far and wish to learn for the rest of this lecture. Give any comment or suggestion on this lecture. (3 point)


