

대학원 최신원격탐사 기말시험  
 강원대학교 지구물리학과 이 훈 열 교수  
 2009년 12월 11일(목) 13:00 ~ 15:50

**Part I: Electromagnetic Theory**

1. Derive the Maxwell's Equations.
2. Describe the meanings of permittivity, conductivity and permeability.
3. Write the constitutive equations of a linear isotropic homogenous material, and describe the Maxwell's equations by using  $\vec{E}$  and  $\vec{H}$  only.
4. Based on the Maxwell's equations in  $\vec{E}$  and  $\vec{H}$  only, derive the differential equations of electromagnetic wave propagation and suggest a solution for it.

**Part II: SAR Focusing**

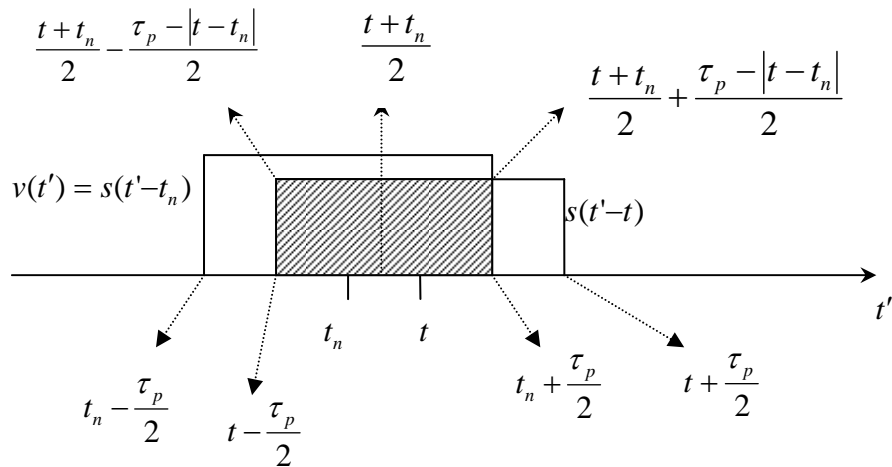
A SAR antenna transmits the following chirp signal,

$$s(t) = \exp[j2\pi(f_c t + Kt^2 / 2)], \quad |t| < \tau_p / 2.$$

1. Define phase  $\phi(t)$ , frequency  $f(t) = \dot{\phi}(t) / 2\pi$ , and chirp bandwidth  $B_R$  of the above signal.

The returned signal from a point target is a delayed chirp signal,  $v(t) = s(t - t_n)$ , where  $t_n = 2R/c$  is the 2-way travel time of the pulse from the sensor to the target. The range compression is performed by the following matched filtering:

$$g(t) = \int_{-\infty}^{\infty} s^*(t' - t)v(t')dt'.$$



2. Prove that the above matched filtering results in

$$g(t) \approx \exp(j2\pi f_c t) \exp(-j4\pi R / \lambda) \frac{\sin[\pi K \tau_p (t - t_n)]}{\pi K (t - t_n)},$$

and find the range time resolution  $\delta t$ .

As the satellite moves forward, many pulses are to be transmitted and backscattered from the point target. Each returned pulse is then range-compressed and sampled at  $t = t_n$ . The carrier structure of the signal is stripped away by the linear operation of complex demodulation. The range-compressed signal of each pulse transmitted at slow time  $s$ , is then (see the figure below)

$$\hat{g}(s) = \exp[j\phi(s)] = \exp[-j4\pi R(s) / \lambda].$$

The Taylor's series expansion of  $R(s)$  centered at  $s = s_c$  (the time at which the target is located at the beam center) is:

$$R(s) = R_c + \dot{R}(s_c)(s - s_c) + \ddot{R}(s_c)(s - s_c)^2 / 2 + \dots$$

3. Using the definition of Doppler frequency:

$$f_D(s) = \dot{\phi}(s) / 2\pi \quad \text{and} \quad \dot{f}_D(s) = \ddot{\phi}(s) / 2\pi, \quad \text{show that}$$

$$\hat{g}(s) \approx \exp(-j4\pi R_c / \lambda) \exp\{j2\pi[f_{Dc}(s - s_c) + f_R(s - s_c)^2 / 2]\}, \quad |s - s_c| < S/2.$$

Here  $S$  is the *coherent integration time*, which means the duration the target is within the view of radar pulse.  $f_{Dc} = f_D(s_c)$  is *Doppler centroid*, and  $f_R = \dot{f}_D(s_c)$  is *Doppler rate*. The above signal can be interpreted as the returned pulse of the linear chirp pulse:

$$h(s) = \exp[j2\pi(f_{Dc}s + f_R s^2 / 2)], \quad |s| < S/2.$$

Therefore, we can compress the signal with respect to  $s$  using matched filtering as:

$$\zeta(s) = \int_{-\infty}^{\infty} h^*(s' - s) \hat{g}(s') ds'.$$

4. Prove that the above integration results in

$$\zeta(s) \approx \exp[j2\pi f_{Dc}(s - s_c)] \exp(-j4\pi R_c / \lambda) \frac{\sin[\pi f_R S (s - s_c)]}{\pi f_R (s - s_c)},$$

and find the *azimuth time resolution* ( $\delta s$ ).

### Part III: SAR Interferometry

A focused SAR image has the signal  $\hat{g} = \exp[-j4\pi R/\lambda]$ , where  $R$  is the distance between the sensor and the target and  $\lambda$  is the wavelength of the transmitted microwave.

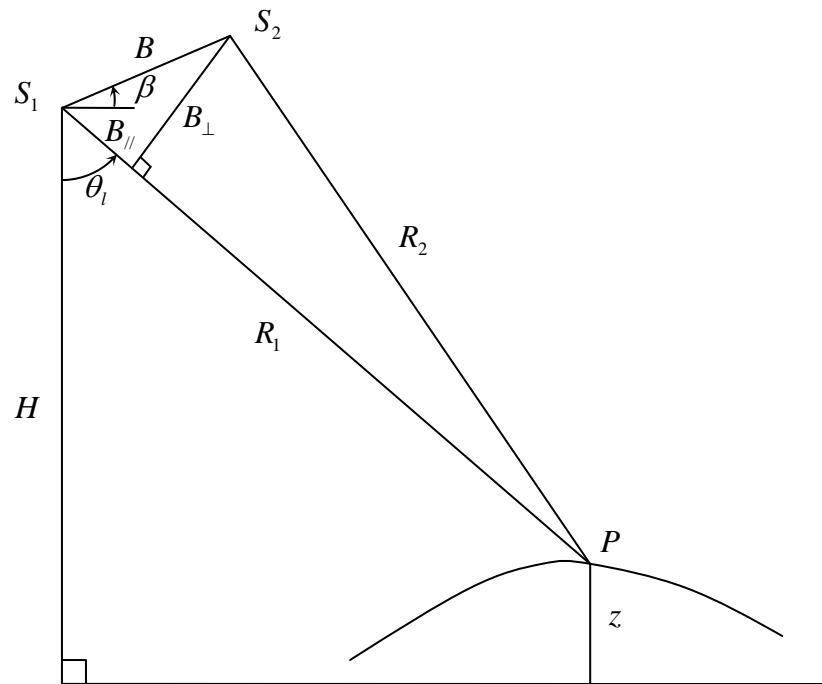
1. What is the phase of the above signal?
2. What is the physical meaning of the phase, including the minus sign.

Consider an interferometric SAR configuration as shown in the figure far below (next page).

3. Describe the phase difference between two SAR observations (interferometric phase),  $\phi = \phi_2 - \phi_1$ , in terms of  $R_1$ ,  $B$ ,  $\theta_1$ , and  $\beta$ , from  $\Delta S_1 P S_2$ .
4. Show that the interferometric phase can be reduced to  $\phi = \frac{4\pi}{\lambda} B \sin(\theta_1 - \beta)$ . You need to know that  $R$  is several hundred kilometers while the baseline  $B$  is no more than several hundred meters. As the ratio  $B/R$  is very small, you can drop  $(B/R)^2$  term during the derivation. Also you need to know that  $\sqrt{1 \pm x} \approx 1 \pm \frac{1}{2}x$  when  $x$  is very small.
5. Prove that  $\phi = \frac{4\pi}{\lambda} B_{//}$ , where  $B_{//}$  the component of the baseline parallel to the radar look direction.
6. Describe the elevation  $z$  of the target  $P$ , in terms of  $H$ ,  $R_1$ , and  $\theta_1$ .
7. Starting from the equations in question 4 and 6, show that the height sensitivity of the interferogram is  $\frac{\partial \phi}{\partial z} = \frac{\partial \phi}{\partial \theta} \frac{\partial \theta}{\partial z} = \frac{4\pi B_{\perp}}{\lambda R_1 \sin \theta_1}$ , where  $B_{\perp}$  is the component of the baseline perpendicular to the radar look direction.
8. Given the phase measurement accuracy of the SAR system is  $\delta\phi_{\text{sys}}$ , find the condition of  $B_{\perp}$  to make the height resolution  $\delta z$  better than the required height resolution  $\delta z_{\text{req}}$ , i.e.,
 
$$\delta z = \frac{\partial z}{\partial \phi} \delta\phi_{\text{sys}} < \delta z_{\text{req}}.$$
9. Starting from the equations in question 4 and 6, find the interferometric phase fringe number, i.e., the number of  $2\pi$  phase fringe in slant range is

$$k_\phi = \frac{1}{2\pi} \frac{\partial \phi}{\partial R_1} = \frac{1}{2\pi} \frac{\partial \phi}{\partial \theta} \frac{\partial \theta}{\partial R_1} \approx \frac{2B_\perp}{\lambda R_1 \tan \theta_i} \quad [\text{m}^{-1}].$$

10. Given the condition that the interferometric phase fringe number should not exceed one fringe over a slant range resolution  $\delta R$ , i.e.,  $k_\phi < \frac{1}{\delta R}$ , limit the  $B_\perp$  to meet this criterion.
11. Combining the limiting conditions of  $B_\perp$  obtained from questions 8 and 10, describe the workable  $B_\perp$  of an InSAR system. Note there are more limiting conditions of  $B_\perp$  than those shown here.



Thank you.