

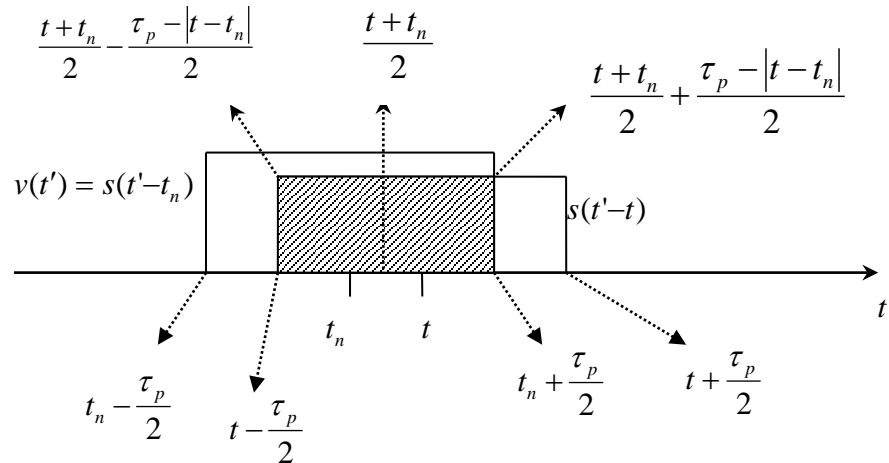
A SAR antenna transmits the following chirp signal,

$$s(t) = \exp[j2\pi(f_c t + Kt^2/2)], \quad |t| < \tau_p/2.$$

1. Define phase $\phi(t)$, frequency $f(t) = \dot{\phi}(t)/2\pi$, and chirp bandwidth B_R of the above signal. (10 points)

The returned signal from a point target is a delayed chirp signal, $v(t) = s(t - t_n)$, where $t_n = 2R/c$ is the 2-way travel time of the pulse from the sensor to the target. The range compression is performed by the following matched filtering:

$$g(t) = \int_{-\infty}^{\infty} s^*(t' - t)v(t')dt'.$$



2. Prove that the above matched filtering results in

$$g(t) \approx \exp(j2\pi f_c t) \exp(-j4\pi R / \lambda) \frac{\sin[\pi K \tau_p (t - t_n)]}{\pi K (t - t_n)},$$

and find the range time resolution δt . (30 points)

As the satellite moves forward, many pulses are to be transmitted and backscattered from the point target. Each returned pulse is then range-compressed and sampled at $t = t_n$. The carrier structure of the signal is stripped away by the linear operation of complex demodulation. The range-compressed signal of each pulse transmitted at slow time s , is then (see the figure below)

$$\hat{g}(s) = \exp[j\phi(s)] = \exp[-j4\pi R(s) / \lambda].$$

The Taylor's series expansion of $R(s)$ centered at $s = s_c$ (the time at which the target is located at the beam center) is:

$$R(s) = R_c + \dot{R}(s_c)(s - s_c) + \ddot{R}(s_c)(s - s_c)^2 / 2 + \dots$$

3. Using the definition of Doppler frequency:

$$f_D(s) = \dot{\phi}(s) / 2\pi \quad \text{and} \quad \dot{f}_D(s) = \ddot{\phi}(s) / 2\pi, \quad \text{show that}$$

$$\hat{g}(s) \approx \exp(-j4\pi R_c / \lambda) \exp\{j2\pi[f_{Dc}(s - s_c) + f_R(s - s_c)^2 / 2]\}, \quad |s - s_c| < S/2. \quad (10 \text{ points})$$

Here S is the *coherent integration time*, which means the duration the target is within the view of radar pulse. $f_{Dc} = f_D(s_c)$ is *Doppler centroid*, and $f_R = \dot{f}_D(s_c)$ is *Doppler rate*. The above signal can be interpreted as the returned pulse of the linear chirp pulse:

$$h(s) = \exp[j2\pi(f_{Dc}s + f_R s^2 / 2)], \quad |s| < S/2.$$

Therefore, we can compress the signal with respect to s using matched filtering as:

$$\zeta(s) = \int_{-\infty}^{\infty} h^*(s' - s) \hat{g}(s') ds'.$$

4. Prove that the above integration results in

$$\zeta(s) \approx \exp[j2\pi f_{Dc}(s - s_c)] \exp(-j4\pi R_c / \lambda) \frac{\sin[\pi f_R S (s - s_c)]}{\pi f_R (s - s_c)},$$

and find the *azimuth time resolution* (δs). (30 points)

5. Describe an ideal, earth-observing remote sensing system. (20 points)

Thank you.